Life of Fred®

Logic

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Polka Dot Publishing

A Note to Readers

The first six chapters of this book make a dandy high school course in logic.

Adding chapters 7–16 makes a full college-level course.

This book is complete. It has many topics that other logic books omit.

✯ Every logic book talks about the five connectives—&(and), ¬(not), ∨(or), ⇒(implies), and ⇔(if and only if)—but few reduce them all down to one connective that can do the job of all five.

✯ Few present 17 fallacies of logic—ultimately convincing six-year-old Fred that he has a wife.

✯ Many present Gödel's Incompleteness Theorems, but few prove the Diagonal lemma that is at the heart of those theorems.

When I'm writing, it is in this typeface. (Times New Roman)
When you, my reader, talk, ***it is in this typeface.*** (***Allegro***)
When Fred is thinking, he prefers AustinsHand.

Puzzles are scattered throughout the book. I hate the thought of "exercises" or "problems." If you want exercise, head to the gym. If you want problems, tell the government you are not going to pay taxes.

Puzzles are meant to feel more like . . . puzzles. Some are super easy, and some might stump your logic teacher.

PREREQUISITES FOR LOGIC

You won't need any algebra, geometry, trig, or calculus. That's the good news.

On the other hand, having hair under your arms (shaved or not) for a couple of years is a fair indication that your brain's reasoning power is developed enough to work with logic.

You won't need a calculator or a protractor or a computer. There are no separate teacher's manuals, answer books, or DVDs. It's all right here.

This book has some giggles . . . like when the Duck walks into Fred's office on the first page.

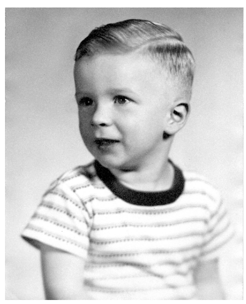
This book is cheap.* No other complete logic book can compete. I, your author, retired from high school and college teaching in 1980 and have no need for gobs of royalty money at this point to pay my light bills.

One last thing . . . hundreds of thousands of readers have asked for a picture of the author. On the back covers of many books, publishers like to stick photos of the author.

Usually, for male authors, the photo shows him standing in the wind with a leather coat—a really rugged guy. For female authors, she will be pictured as every man's sweetheart.

The publisher had decided that no *Life of Fred* book will have Stan's picture on the cover. He explained, "It might hurt sales."

However, he can't stop me from including a photo inside the book. Here is my photo taken several years ago. This really is me!



Dr. Schmidt

With my best wishes,
Stan

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*The publisher wanted me to write *inexpensive*, but I like *cheap* better.

Contents

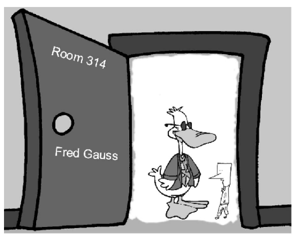
Chapter One
Sentences

A scratch at the door. It was almost like someone was rubbing feathers against the door. Fred looked up from his logic lecture notes that he had been working on. He hopped off his chair and headed to the door.

"I wonder who that could be," he said to his doll Kingie. "We usually don't have students visiting my office on Saturday afternoon."

Kingie shrugged his shoulders and continued working on his oil painting.

More scratching. Fred opened the door. His heart sank to the floor. It was the Duck.



The last time Fred had seen the Duck was a year ago when Fred was 5.* It had been a very bad experience for Fred.

Fred tried to never judge someone by their appearance. There could be a million reasons why Duck was wearing a tie, a sports coat, and sunglasses. The fact that Duck was 4½ feet tall and Fred was only 3 feet tall wasn't that important.

What did bother Fred were the sentences that Duck uttered:

"Good morning." (It was 2 p.m.)

"This is a lovely barn with lots of cows." (Kingie didn't like being called a cow, and this was Fred's math office. It wasn't a barn.)

"You've grown a lot since the last time I saw you." (Fred has been 36 inches tall and 37 pounds for a long time. It was the Duck that had grown six inches in the last year.)

Every time Duck spoke, he lied. Every sentence was false.

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*In the very first book of the Elementary Series: *Life of Fred: Apples.*

When Fred was 5, this really bothered him. In fact, back in *Apples* Fred ran away to escape from Duck. He couldn't stand hearing lie after lie.

Now at the age of 6, Fred's feelings toward Duck had changed. "Please come in and sit down."

Duck came in and said, "I'd rather stand," and then he sat down.

◊ ◊ ◊

Stop! Wait a minute. I, your reader of this book, have a question. I am grateful that you, Mr. Author, allow me to interrupt. Most other authors jabber and permit no one to ask for clarification.

What did you, my reader, have in mind?

Why in blazes did Fred invite Duck in? I would have turned him around and kicked him in the tail feathers. Duck just utters pure lies. If I wanted that, I would just turn on the television news.

There are three reasons why Fred welcomed Duck instead of kicking him out. ① Duck is 50% taller than Fred.* If you were six feet tall, that would be like messing with someone who is nine feet tall. That's NAGI (Not A Good Idea). ② From the third sentence at the start of this chapter, we note that Fred is working on his logic lecture notes. In his logic class on Monday—today is Saturday—Fred would love to bring Duck with him. Duck is a good example of *sentences* in logic. ③ Duck is a Fountain of Truth—I'll tell you about that later. Right now, I want to concentrate on ② and what *sentences* in logic are.

Go ahead. Who's stopping you?

Um. Didn't you interrupt me?

Gulp. Sorry. Go on with your story.

◊ ◊ ◊

Fred explained, "Sentences in logic are different than sentences in English. Sentences in logic must be either true or false."

important!

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*Duck is 4½ feet tall. That's 4.5 feet. Fred is 3 feet tall. Back from old *Decimals and Percents* days, if you wanted to compute "50% more than," you start with 100% and tack on another 50%. Then 50% taller than 3 feet means 150% of 3 feet. In decimals this means 1.5 × 3, which is 4.5.

Kingie set down his paint brush and complained, "What makes Duck so special? It's unfair that you are thinking of taking him to your logic class and not me. Humpf! Take me!"*

Fred smiled. "It's simple. Every sentence Duck says is a sentence in logic—it is either true or false. Every sentence you have just said is not a logic sentence.

✓ "Your first sentence—What makes Duck so special?—is a question.

✓ "Your second sentence—It's unfair . . .—is an opinion.

✓ "Your third sentence—Humpf!—is an interjection.

✓ "Your fourth sentence—Take me!—is a command, which in some English classes is called an imperative.

"None of your English sentences is a logic sentence."**

Duck wanted to show off and said:

Kingie is a ten-pound elephant.
All rabbits are white.
There is a state in the U.S.A. that begins with the letter B.

These are three sentences in logic. Fred wrote them down in his logic notes and labeled them K, R, and B.

> *Every sentence in logic is abbreviated with a capital letter.*

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*Small quick explanation for those readers who are new to the *Life of Fred* series: Kingie is Fred's doll. They have known each other for almost all of Fred's life. When Fred was four days old, the man at King of French Fries gave this doll to Fred. That's why Fred named his doll Kingie.

Fred didn't have to tell Kingie that he was thinking of taking Duck to his logic class on Monday, because, as everyone knows, dolls can read their owners' minds.

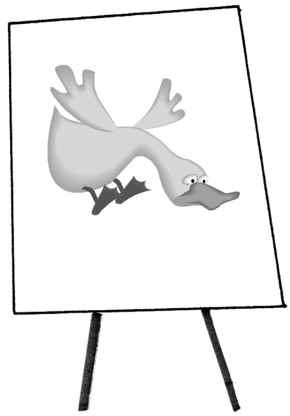
**In English when you have several paragraphs quoting the same person, only the last paragraph has the close quote (") symbol.

And while we're doing English, please note that we don't write, "None of your English sentences *are*. . . ." The subject of the sentence is *none*, and so the verb is singular—*is,* not *are*.

Kingie was not in a good mood. In the past, when Fred had a cat, dogs, or a llama as pets, they were all bad news for Kingie. Now Fred was bringing a lying duck into his office. Kingie muttered, "Logic is nuts. (opinion) What if you have 27 logic sentences? (question) You are going to run out of letters! (a real logic sentence)" Kingie wanted to have the "last word" before he headed back to painting.

Duck repeated Kingie, "You are going to run out of letters!"

Kingie was fuming. He understood what it means when a duck that always lies *agreed* with him? Kingie put a new canvas on his easel and painted the ugliest duck he could.

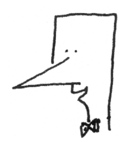


Fred didn't know what to do in this emotionally charged situation. He knew that on Monday when he introduced the sentence letters ABCDEFGHIJKLMNOPQRSTUVWXYZ, that someone might ask about the case in which you have 27 logic sentences.

He would first say that it was quite rare in logic to have more than three sentences to play with. In most cases, logicians liked to use A, B, and C, or P, Q, and R.

Years ago when Fred was first teaching logic, he used to say that if you had more than 26 logic sentences, you could use letters from other alphabets: or БДЖИЛЦЯЮЂЖ, or אבדהחלמסעץקש. But some student would then ask, "Well, what if you had a thousand logic sentences? You would run out of different alphabets."

Fred was older (age 6) and wiser now. When asked that question on Monday, he would say that logicians had solved that puzzle. If they needed a lot of logic sentences, instead of using ABCDEFGHIJKLMNOPQRSTUVWXYZ, they use P_1, P_2, P_3, P_4. . . . Logicians have an infinite number of sentence letters.



It's tough to win a math argument with Fred.

Fred liked the fact that Duck could only utter logic sentences, but that's not what intrigued Fred. He realized that Duck was a Fountain of Truth.

That's nuts! I, your reader, know that's the sun and not a cheeseburger. How in the world can Fred think that this lying Duck is a Fountain of Truth ***as he calls it?***

> *Note to readers:*
> *Answers to all of the puzzles are given in the back of this book.*
> *Please do not just read the question and turn to the answer. That would be like going to a gym and just watching people work out.*

Puzzle #1: If you want to find out from Duck the truth about something, you first have to learn whether Duck is knowledgeable about that topic. For example, if you want to know whether the Goldbach conjecture is true, you can be fairly certain that Duck doesn't know the answer.*

KITTENS University is where Fred teaches. It is in Kansas. Suppose that Fred wanted to find out *whether Duck knew* what the capital of Kansas is. What question could Fred ask Duck?

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*That's because no one (today) knows the answer.

Back in 1742 Christian Goldbach had two conjectures. The first was that every even integer greater than 2 can be written as the sum of two primes. 4 = 2 + 2 6 = 3 + 3 8 = 5 + 3 10 = 7 + 3 and so on.

His second conjecture was the every integer greater than 5 can be written as the sum of three primes. 6 = 2 + 2 + 2 7 = 2 + 2 + 3 8 = 2 + 3 + 3 and so on.

For over 300 years no one has been able to figure out whether either of these conjectures is true.

Once you have determined that Duck knows what the capital of Kansas is, the next step is to force this lying duck to become a Fountain of Truth.

You can't just ask Duck, "What is the capital of Kansas?" He will lie and say that Hot Dog is the capital.

If you don't know what the capital of Kansas is, you can't just keep making guesses: Is Sacramento the capital of Kansas? Is Bismark the capital of Kansas? Is Berlin the capital of Kansas? That would take forever.

Puzzle #88: [harder] What question could you ask Duck in order to force him to tell you that Topeka is the capital of Kansas? (The puzzles are not numbered consecutively so that you won't accidently see the answer to this question when you read the answer to Puzzle #1.)

Note to reader:

In many parts of math—such as algebra—it is important that know the current material before you head on to the next chapter. For example, if you don't know how to factor trinomials such as $x^2 - 5x + 6$, *then solving* $(x^2 - 5x + 6)(x^2 + 6x + 8) = 0$ *can be a real pain.*

In contrast, in this course in logic, these puzzles can be a source of pleasure for a week. It might be next Monday before you figure out how to squeeze "Topeka" out of Duck.

The Two Halves of Logic

Pure Logic

Logic has a language. We will call it L.

So far, we know that L contains sentence letters such as A, B, C, or P_1, P_2, P_3, P_4.

Applied Logic

Here is where we *use* that logic language L.

We create a **model** for L in which every sentence letter has a meaning. This is called **semantics**.

B might stand for "Betty is a Ph.D. student at KITTENS.

C might stand for "Fred is a canary."

One model for L can be set theory, which we will look at in Chapter 8.

Another model for L can be arithmetic, which we will look at in Chapter 9.

In fact, virtually all parts of math (including geometry) can be thought of as models of L.

Models of L are sometimes called **structures** for L.

Memory Aid Tie these four words together:

model,

meaning,

semantics, and

structure.

They are all part of applied logic.

Duck is a semantical kind of bird. All we get out of him are models for L. For example:

A is Apples sing in the moonlight.
B is Bowling balls are my favorite candy.
C is Cinderella likes eating toothpaste on toast.

In pure logic, in language L, we just have sentence letters like P, Q, and R. It would be silly to ask, "Is P true?"

It is only when we assign a structure to L, that we can discuss whether P is true or false.

When Fred teaches logic, he has two favorite models* he likes to talk about: set theory and arithmetic.

Hold it! Stop the show! I, your reader, am starting to panic. I don't care if Fred messes with arithmetic. I can handle 2 + 3 = 5, but it's been a hundred years since I've done any set theory. The only thing I can remember is that a set is any collection of objects. That's it. Before you, Mr. Author, go any further, tell me everything about set theory.

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*models = structures

Everything? Geep! There are *books* written about set theory. There are mathematicians who spend their whole lives just playing with sets. I can't . . .

I don't mean everything about set theory. How about just a bit of a refresher. Just the super basics. I bought this book, and I want you to follow my wishes.

I thought authors were supposed to figure out what goes in a book.

This is a brave new world. Without me, you are nothing.

Wow. That sounds like solipsism. Some of my readers already know about sets. I'll put the basics about sets in a box on this page. Then those other readers can skip over the box if they want to.

It's a deal.

Handy Short Course in Set Theory

A set is any collection.

{⌚, ❀, ✈} is a set that contains three members.

"{" and "}" are called braces. You stick braces around a set.

∈ means "is a member of." ❀ ∈ {⌚, ❀, ✈}

∉ means "is not a member of." ☎ ∉ {⌚, ❀, ✈}

ℕ is the set of natural numbers {1, 2, 3, 4, 5, . . .}.

¾ ∉ ℕ

Two sets are equal if they have exactly the same members.

If A and B are equal sets, then we write A = B. This is a different "=" than the one used in arithmetic.

Another word for *members* is *elements.*

The world's smallest set is { }, which is called the empty set. It has no elements in it.

The cardinality of a set is the number of elements in the set. The cardinality of {⌚, ❀, ✈} is 3. The cardinality of { } is 0.

That amount of set theory should hold us for now. On Monday, if Fred wanted to create a set theory structure for L, all he would need to do is assign each sentence letter of L to some true-or-false sentence in set theory.

Show me!

Fred could assign A to ❀ ∈ {⌚, ❀, ✈}.
He could assign B to ❀ ∉ {⌚, ❀, ✈}.
He could assign C to {⌚, ❀, ✈} = {⌚, ❀, ✈}.

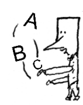
Ha! I caught you. There are an infinite number of sentence letters in L. ***There are*** C, D, E, . . . , ***and there is the infinite list*** P_1, P_2, P_3, P_4. ***He could spend all Monday and never finish making his model.***

You didn't let me finish. I was going to say that Fred would then assign all other sentence letters of L to ✈ ∈ {⌚, ❀, ✈}.

Often in making a model, we will only need to assign some of the sentence letters. The rest of the sentence letters can be assigned to anything in the model. It is a little like throwing the unneeded sentence letters in the trash can.

Puzzle #40: Create an *arithmetic* model for L.
Here's a start. Assign A to ¼ + ⅔ = 27.

I, your reader, hate to interrupt again*, but this pure logic thing seems so stupid. All you have are sentence letters—like A, B, or C—and there's nothing to do with them. They can't even be true or false in pure logic.

Fred enjoys juggling them. 

Before you interrupt again, I would like to point out that this is only Chapter 1 in which I've introduced sentence letters. Once we get to Chapter 2 and introduce connectives, things will get much more interesting in pure logic.

How soon do we get to Chapter 2? I'm ready!

Just turn the page.

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*This is obviously a lie. Duck is having a bad influence on you. You seem to enjoy the freedom you have to interrupt me.

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in the
Life of Fred series

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